Quiz 6 Solution

February 12, 2018

1. (2 points) Find the derivative of $y = \ln(\sin x - \sec x) + \sqrt{x}$ at $x = \dfrac{\pi}{3}$.

   Solution: We can't use log rules to simplify, so we use chain rule to take the derivative:

$$\begin{aligned} y' &= \frac{d}{dx}[\ln(\sin x - \sec x)] + \frac{d}{dx}[\sqrt{x}] \\ &= \frac{1}{\sin x - \sec x} \cdot \frac{d}{dx}[\sin x - \sec x] + \frac{1}{2}x^{-1/2} \\ &= \frac{\cos x - \sec x \tan x}{\sin x - \sec x} + \frac{1}{2\sqrt{x}} \end{aligned}$$

$$\text{So } y'\left(\frac{\pi}{3}\right) = \frac{\cos\left(\frac{\pi}{3}\right) - \sec\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{3}\right)}{\sin\left(\frac{\pi}{3}\right) - \sec\left(\frac{\pi}{3}\right)} + \frac{1}{2\sqrt{\left(\frac{\pi}{3}\right)}}$$

$$= \frac{\frac{1}{2} - 2 \cdot \sqrt{3}}{\frac{\sqrt{3}}{2} - 2} + \frac{\sqrt{3}}{2\sqrt{\pi}}$$

$$= \frac{1 - 4\sqrt{3}}{\sqrt{3} - 4} + \frac{\sqrt{3}}{2\sqrt{\pi}} \text{ (by multiplying the first quotient by } \frac{2}{2})$$

   Answer: $\dfrac{1 - 4\sqrt{3}}{\sqrt{3} - 4} + \dfrac{\sqrt{3}}{2\sqrt{\pi}}$

2. (2 points) Find the derivative of $y = e^{\csc(2t+7)}$.

   Solution: We use chain rule with outside function e^u and inside function $\csc(2t+7)$:

$$\begin{aligned} y' &= e^{\csc(2t+7)} \cdot \frac{d}{dt}[\csc(2t+7)] \\ &= e^{\csc(2t+7)} \left(-\csc(2t+7)\cot(2t+7) \cdot \frac{d}{dt}[2t+7] \right) \\ &\quad \text{(by chain rule with outside function } \csc u \text{ and inside function } 2t+7) \\ &= -2e^{\csc(2t+7)} \csc(2t+7)\cot(2t+7) \end{aligned}$$

   Answer: $-2e^{\csc(2t+7)} \csc(2t+7)\cot(2t+7)$

3. (1 point) Are you planning to take MA 16020?

   Answer: Yes or No.